Sec. 12.9 Linear Programming

Linear Programming Problem in two variables x and y consists of maximizing (or minimizing) a linear objective function $z = Ax + By$ where A and B are real numbers, not both 0, subject to certain conditions or constraints, expressible as linear inequalities in x and y.

Constraints – conditions that can be written as linear inequalities

Objective function – must be a linear expression in $Ax + By$ form

Ex: A retired couple has up to $25,000 to invest. As their financial advisor, you recommend that they place at least $15,000 in Treasury bills yielding 6% and at most $5000 in corporate bonds yielding 9%. How much money should be placed in each investment so that income is maximized?

x = amount invested in Treasury bills
y = amount invested in Corporate bonds

Objective Function: $I = .06x + .09y$

Constraints: $x \geq 0$ $y \geq 0$ (must be positive)
$x \geq 15$ (at least 15,000)
$y \leq 5$ (at most 5000)

Feasible point – each point (x, y) that satisfies the system of linear inequalities (constraints), then choose the one that maximizes or minimizes

Solution – the feasible point or points that maximize or minimize the objective function along with the corresponding value of the objective function

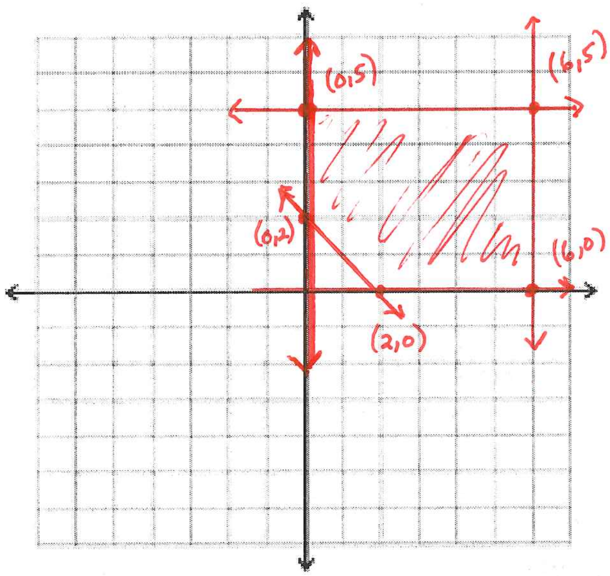
To find:

1. If a linear programming problem has a solution, it is located at a corner point of the graph of the feasible points.
2. If a linear programming problem has multiple solutions, at least one of them is located at a corner point of the graph of the feasible points.
3. In either case, the corresponding value of the objective function is unique.

To Solve a Linear Programming Problem:

1. Write an expression to the quantity to be maximized or minimized. This expression is the objective function.
2. Write all the constraints as a system of linear inequalities and graph the system.
3. List the corner points of the graph of the feasible points.
4. List the corresponding values of the objective function at each corner point. The largest (or smallest) of these is the solution.

Ex: Minimize the expression z = 2x + 3y subject to the constraints: $\begin{cases} y \le 5 \\ x \le 6 \\ x + y \ge 2 \\ x \ge 0 \\ y \ge 0 \end{cases}$

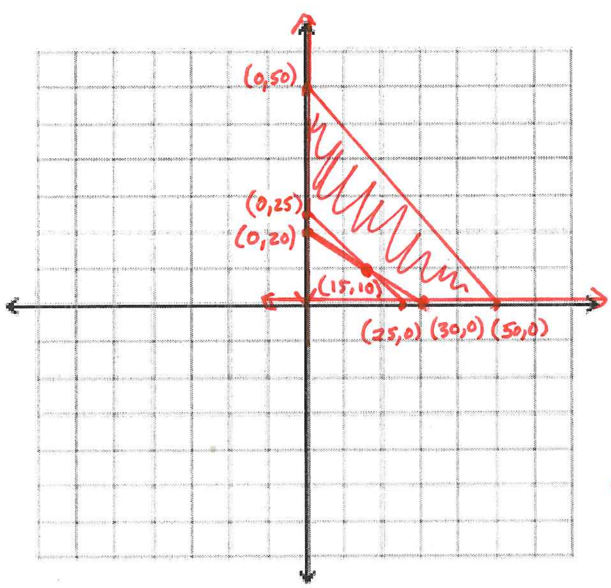


Corner Point	Value of Objective Function
(0,2)	2(0)+3(2) = 6
(0,5)	2(0)+3(5) = 15
(6,5)	2(6)+3(5) = 27
(6,0)	2(6)+3(0) = 12
(2,0)	2(2)+3(0) = 4

Minimum value of 4 at (2,0)

Ex: An owner of a fruit orchard hires a crew of workers to prune at least 25 of his 50 fruit trees. Each newer tree requires 1 hour to prune, while each older tree needs 1.5 hours. The crew contracts to work for at least 30 hours and charge $15 for each newer tree and $20 for each older tree. To minimize his cost, how many of each kind of tree will the orchard owner have pruned? What will the cost be?

x = number of newer trees y = number of older trees



OBJECTIVE FUNCTION: $C = 15X + 20y$

$X \ge 0$
$y \ge 0$
$X + 1.5y \ge 30$
$X + y \ge 25$
$X + y \le 50$

SOLVE SYSTEM TO FIND POINT OF INTERSECTION

$X = 30 - 1.5y$
$30 - 1.5y + y = 25$
$30 - .5y = 25$
$-.5y = -5$
$y = 10$
$X + 10 = 25$
$X = 15$

CORNER POINTS

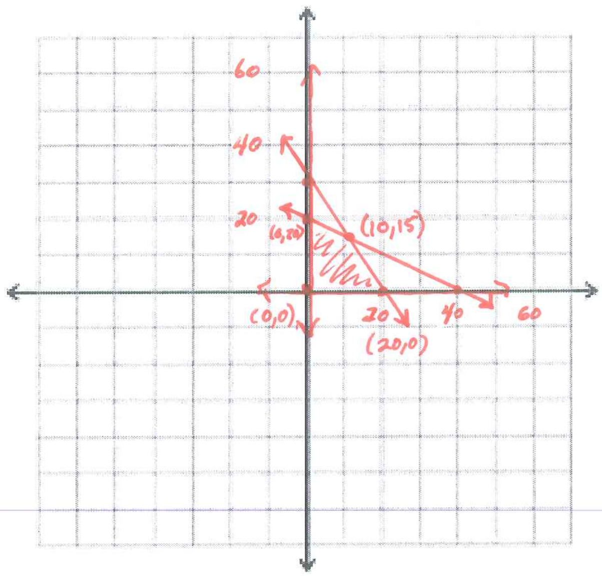
(0,25) 15(0)+20(25) = 500
(15,10) (15)(10)+20(10) = 350 (MIN)
(30,0) (15)(30)+20(0) = 450
NOT CONSIDERED (50,0)
(0,50)

TO MINIMIZE COST PRUNE 15 NEW AND 10 OLD TREES

Ex: A factory manufactures two kinds of ice skates: racing skates and figure skates. The racing skates require 6 work hours in the fabrication department, whereas the figure skates require 4 work hours there. The racing skates require 1 hour in the finishing department, whereas the figure skates require 2 work hours there. The fabricating department has available at most 120 work hours per day and the finishing department has not more than 40 work hours per day available. If the profit on each racing skate is $10 and the profit on each figure skate is $12, how many of each should be manufactured each day to maximize profit?

x = # of racing skates y = # of figure skates

OBJECTIVE FUNCTION: $P = 10x + 12y$



$$\left.\begin{aligned} 6x + 4y &\le 120 \\ x + 2y &\le 40 \end{aligned}\right\}$$ Solve System to Find Intersection

$x \ge 0 \quad y \ge 0$

$x = 40 - 2y$

$6(40 - 2y) + 4y = 120$

$240 - 12y + 4y = 120$

$240 - 8y = 120$

$-8y = -120$

$y = 15$

$x = 40 - 2(15)$

$x = 10$

$(0, 20)$	$0(10) + 12(20) = 240$
$(20, 0)$	$20(10) + 12(0) = 200$
$(10, 15)$	$10(10) + 12(15) = 280$ MAX

Profit is maximized at $280 when 10 racing skates and 15 figure skates are made.

HOMEWORK: Day 1: pg 840 #1-6,7,9,12
Day 2: pg 840-842 #19, 23, 27, 28, 30